Some Notes on the Skwirzynski Transformation

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These notes are intended to be an extension to *Chapter 2.8.2 The 'Skwirzynski' transformation*, by R. Nouta. In here, the importance of the Skwirzynski transformation when designing realizable even order ladder filter structures with lumped elements has been described.

The description of the transfer function of an even order low pass filter –resulting from the theory– is commonly denoted as a type A design. Realizable filters will be of Type B or Type C.
Type B corresponds to Nouta's **Example 3, Possibility 1**, while Type C is described in **Example 3, Possibility 2**.
Skwirzynski denotes a Type B transformation as the '**wiggle**' transformation, indicated with $\Omega \rightarrow \widetilde{\Omega}$.
In his terminology, a Type C transformation is a '**bar**' transformation: $\Omega \rightarrow \overline{\Omega}$.

Type A

For illustration, a stylized transfer curve of a 6th order normalized Cauer filter is used with the characteristic frequency points denoted in Figure 1. Note the specific numbering of the Ω_p versus the Ω_s frequency points, which indicate the relationship between equally numbered points.

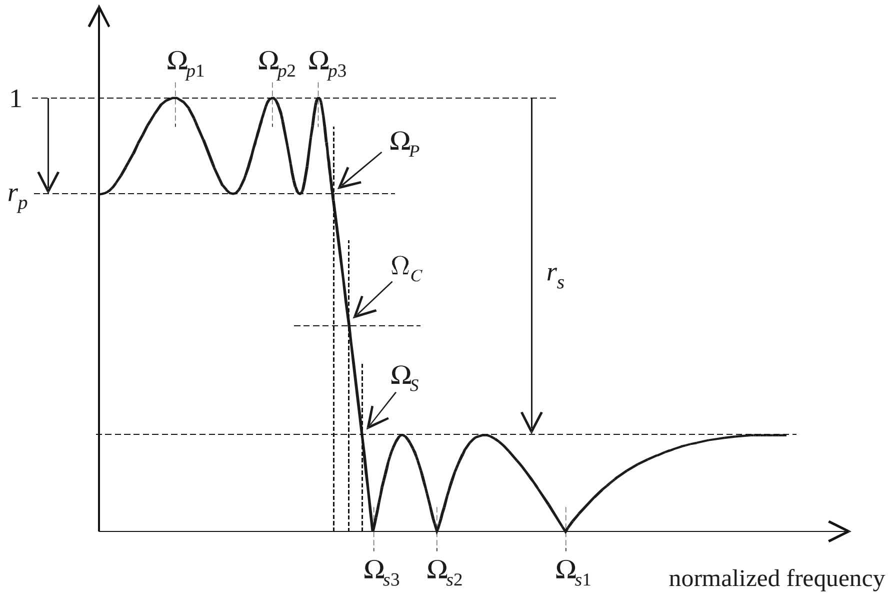

Figure 1.

In the literature we encounter two frequently used, different choices for normalization

1. method 1, for reasons of symmetry (used a.o. by Nouta, Skwirzynski and Antoniuou):
   $\Omega_P = \sqrt{k}, \quad \Omega_C = 1$ and $\Omega_S = \dfrac{1}{\sqrt{k}} \quad (k < 1)$
2. method 2, for reasons of comparability with the Chebyshev normalization (used by Christian & Eisenmann):
   $\Omega_P = 1, \quad \Omega_C = \dfrac{1}{\sqrt{k}}$ and $\Omega_S = \dfrac{1}{k} \quad (k < 1)$

Type B

A new frequency variable ω will be used that is obtained by shifting Ω_{s1} to infinity ($\omega_{p1} = \infty$), while keeping the frequency points at 0 and 1 in place as illustrated in Figure 2.

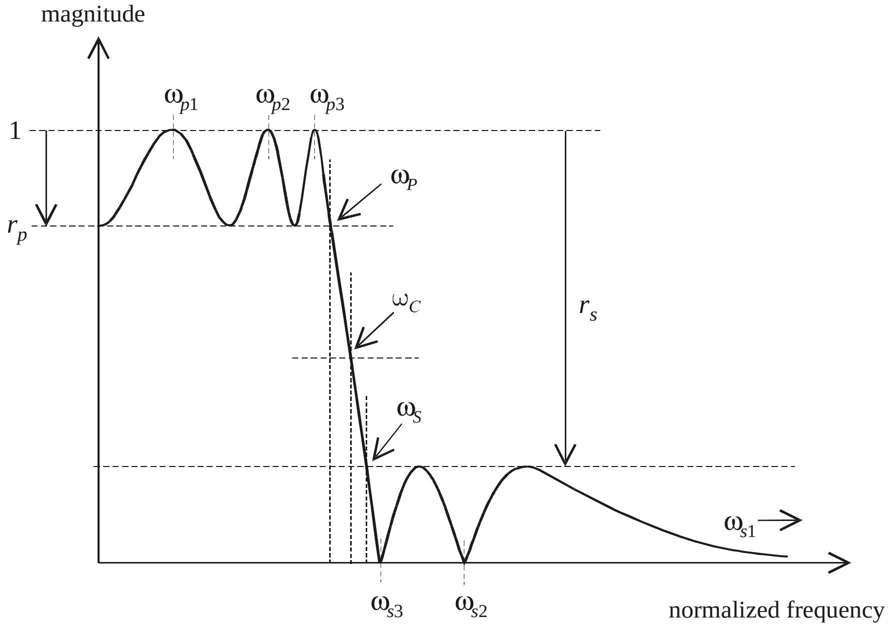

Figure 2.

Now the new frequency variable can be expressed in terms of the old one with

$$\omega = \Omega \sqrt{\frac{\Omega_{s1}^2 - 1}{\Omega_{s1}^2 - \Omega^2}}$$

If P are the complex poles of the transfer function, then the new poles p can be obtained with

$$p = P \sqrt{\frac{\Omega_{s1}^2 - 1}{\Omega_{s1}^2 + P^2}}$$

Since usually Ω_{s1} has been derived from Ω_{p1}, it will be more accurate to use just this Ω_{p1}.Considering the two normalization methods, we can write

$$\boxed{\omega = \Omega \sqrt{\frac{1 - C_k \Omega_{p1}^2}{1 - C_k \Omega_{p1}^2 \Omega^2}} \quad \text{and} \quad p = P \sqrt{\frac{1 - C_k \Omega_{p1}^2}{1 + C_k \Omega_{p1}^2 P^2}}}$$

where

	normalization	
	method 1	method 2
C_k	1	k^2

since in method 1 $\Omega_{s1} = \dfrac{1}{\Omega_{p1}}$, while in method 2 $\Omega_{s1} = \dfrac{1}{k\Omega_{p1}}$.

Type C

Except for the shift of Ω_{s1} to infinity, Ω_{p1}will be shifted to 0 as well.

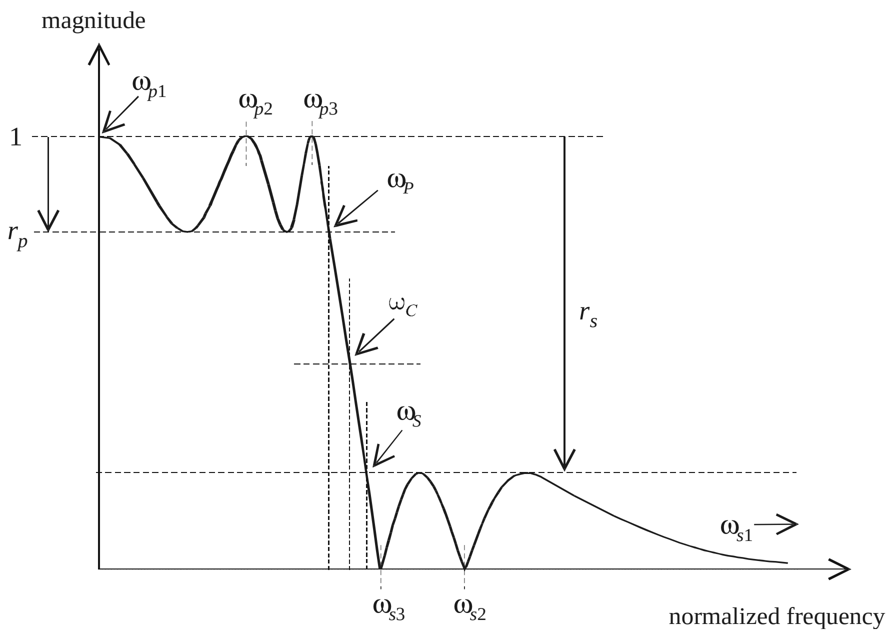

Figure 3.

The new frequency variable becomes

$$\omega = \sqrt{\frac{\left(\Omega_{s1}^2 - 1\right)\left(\Omega^2 - \Omega_{p1}^2\right)}{\left(1 - \Omega_{p1}^2\right)\left(\Omega_{s1}^2 - \Omega^2\right)}}$$

and the new locations of the poles become

$$p = -\sqrt{\frac{\left(\Omega_{s1}^2 - 1\right)\left(P^2 + \Omega_{p1}^2\right)}{\left(1 - \Omega_{p1}^2\right)\left(\Omega_{s1}^2 + P^2\right)}}$$

Again realizing that Ω_{s1} has in fact been derived from Ω_{p1}, we will eliminate Ω_{s1}. With the different normalization methods in mind, we write

$$\boxed{\omega = C_m\sqrt{\frac{\Omega^2 - \Omega_{p1}^2}{1 - C_k\Omega_{p1}^2\Omega^2}} \qquad \text{and} \qquad p = -C_m\sqrt{\frac{P^2 + \Omega_{p1}^2}{1 + C_k\Omega_{p1}^2 P^2}}}$$

where

	normalization method 1	normalization method 2
C_m	1	$\sqrt{\dfrac{1 - k^2\Omega_{p1}^2}{1 - \Omega_{p1}^2}}$
C_k	1	k^2

The equations above have been implemented and tested in several MATLAB programs, viz.

`d:\filters\cauer\skwirBC.m` (first test program),

`d:\filters\cauer\data6ABC.m` (comparison of the output data of 6th order filters),

and the final function

`d:\filters\cauer\Cauer.m`

with syntax

```
[Ws,wp,ws,p] = Cauer(N,rp,rs,normth<,ftype>)
```

where

input parameters

`N`	:	the order of the filter
`rp`	:	the maximum ripple in the pass band in dB
`rs`	:	the minimum attenuation in the stop band in dB
`normth`	:	the normalization method, 0 for $\Omega_C = 1$ or 1 for $\Omega_P = 1$
`ftype`	:	for even order filters, type 'A', 'B' or 'C' needs to be specified

output parameters

`Ws`	:	Ω_S or ω_S
`wp`	:	the Ω_p or ω_p vector
`ws`	:	the Ω_s or ω_s vector
`p`	:	the complex poles of the Cauer function

Note: `wp` corresponds with the ATTEN.ZEROS, and `ws` with the ATTEN.POLES listed in Christian & Eisenmann

References:

E. Christian & E. Eisenmann, *Filter Design Tables and Graphs*, John Wiley & Sons, Inc. 1966

R. Nouta, *Diktaat Signaalbewerking Deel 1*, November 2002

J.K. Skwirzynski, *Design Theory and Data for Electrical Filters*, Van Nostrand Company Ltd, 1965